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# Uncertainties in the Neutron Coincidence Counting Analyses:

Accounting for Correlation and Uncertainties

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CCS-6

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# Overview

- Correlation in Neutron Multiplicity
- Uncertainty in Neutron Multiplicity
- Curve Fitting
- Statistical Challenges in Neutron Multiplicity
- Correlated Error and Variance Estimates
- The Covariance Matrix for Data Gated at Different Widths
- References

# Neutron Multiplicity Correlation Issue

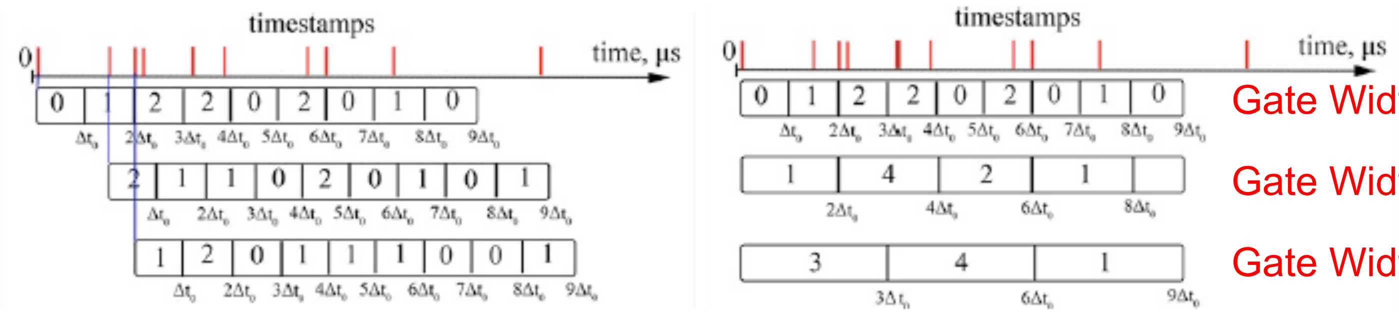
- One Neutron Chain
- Combine Single Gate Width To Get Multiple Gate Widths
- Independence Assumption Violated
- Example of Gate Width Sizes

# Summarizing the Data for the Rossi-alpha and Feynman Methods

Gate Width 1,2,3

Gate Width 1,2,3

Gate Width 1,2,3




Gate Width 1

Gate Width 2

Gate Width 3

Figure 3: Procedures of data processing for traditional Rossi-alpha (l.h.s) and Feynman-alpha (r.h.s.) measurements.

Chernikova, D., Naeem, S. F., Trnjanin, N., Axell, K., & Nordlund, A. (2015).
“The authors want to thank Prof. David Wehe, Prof. Imre Pa´zsit, Prof. Martyn T. Swinhoe and Andrea Favalli for useful discussions and advice.”

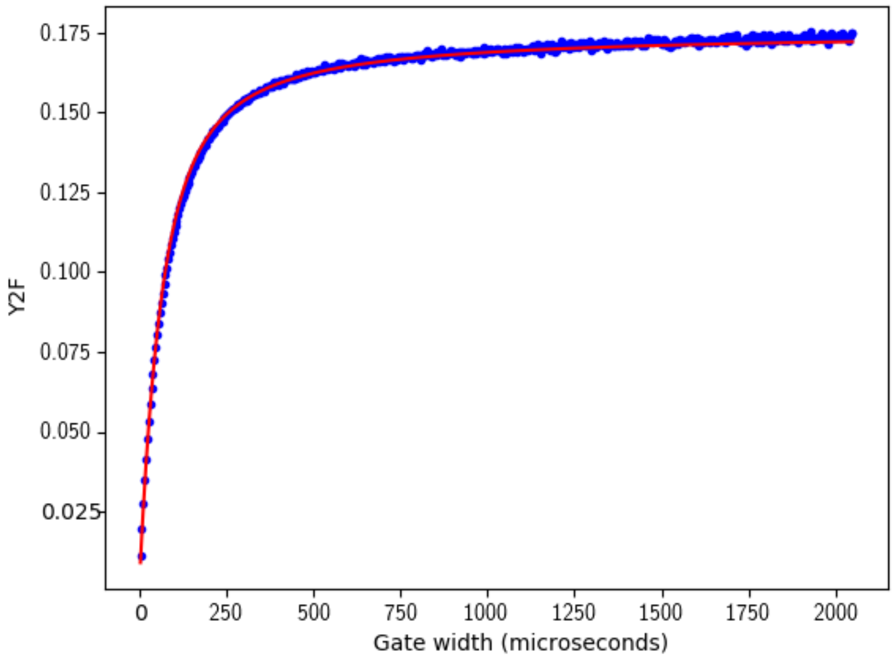
## Where Does the Uncertainty Arise?

- Internal to the Model
  - Segment
- External to the Model
  - Bias
  - Model Misspecification
  - Change in Unaccounted for Factors (Humidity, Geometry, etc.)

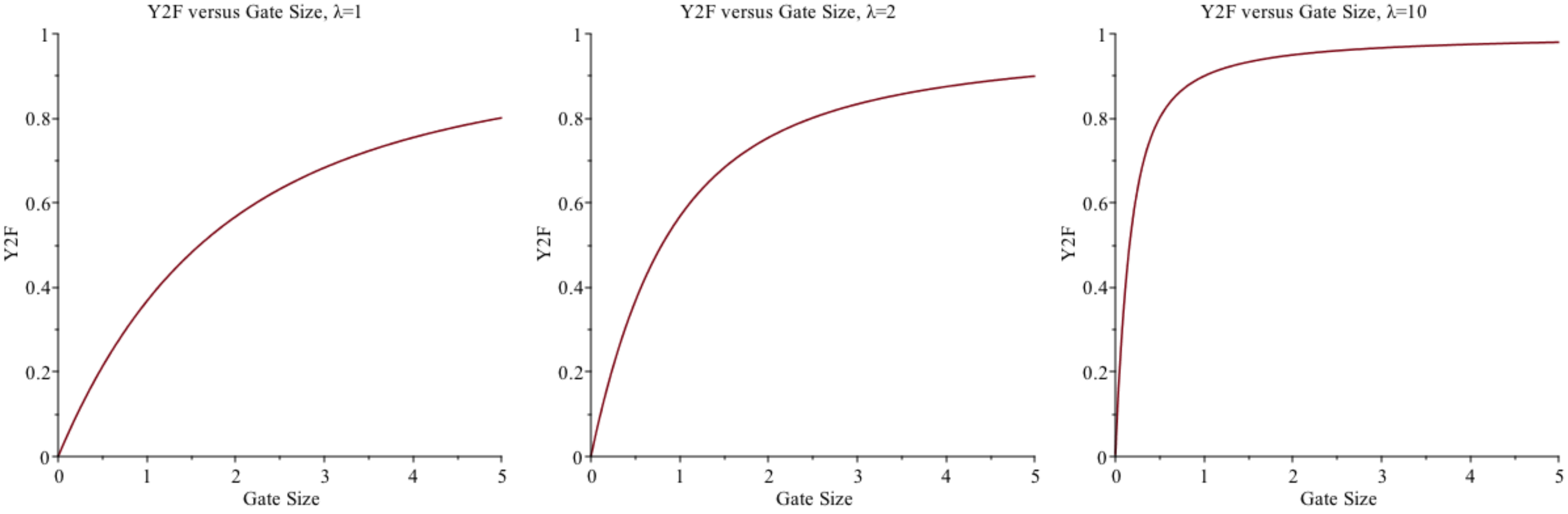
# Multi-gate method

- **Fit Y2 vs gate to determine lambda and R2**
- **Correlations between time gates**

| | |
|---|---|
| Gate fraction | $\omega_2 = 1 - \frac{1-e^{-\lambda\tau}}{\lambda\tau}$ |
| Measured doubles | $Y_2 = \frac{m_2 - \frac{1}{2}m_1^2}{\tau}$ <br> $Y_2 = R_2\omega_2$ |



# Curve Fitting $\lambda = 1, 2, 10,$



# Fitting the Curve

- Fit Y2 vs gate curve with all data points by minimizing the Chi2 in LANL W matrix paper (weighted by counting statistics error)
- Fitting function is (R2/R1) x (1 - (1-exp(-$\lambda$ x $\tau$) / ($\lambda$ / $\tau$)))
- Compute residuals of data points against the fitted curve
- Compute variance of residuals
- Compute autocorrelation of residuals (Equation 2.1.6 in the Box Jenkins book using lag = 1)
- Divide the variance of the residual by $(1 - \rho^2)$, this is now the approximate variance of the predicted fit values
- Y2 (or fitted Y2) and $\lambda$ do not have the same uncertainty

# How to Estimate Uncertainty?

- Classic Error Propagation
- Probability Considerations
  - Counting Statistics
  - Observed minus Expected

# Types of Error

- Count Statistics
- Additive
- Multiplicative
- Data Transformations (Model Misspecification)

# Transformation to a linear model https://en.wikipedia.org/wiki/Non-linear_least_squares

- Untransformed Model: $f(x, \alpha, \beta) = \alpha\ e^{\beta x}$
- Transformed Model: $g(x, \alpha, \beta) = \ln(\alpha) + \beta x$
- "This procedure should be avoided unless the errors are multiplicative and log-normally distributed because it can give misleading results. This comes from the fact that whatever the experimental errors on **y** might be, the errors on **log y** are different. Therefore, when the transformed sum of squares is minimized different results will be obtained both for the parameter values and their calculated standard deviations. However, with multiplicative errors that are log-normally distributed, this procedure gives unbiased and consistent parameter estimates."

# Statistical Challenges

- Error Specification in the Model
- Independence Assumption Violated
  - Correlated Data – $(1 - \rho^2)$ Factor Optimized
- Transformations

# Correlated Error

- See page 50 of Time Series Analysis: Forecasting and Control by Box and Jenkins equations 3.2.13 and 3.214.
- If the data are correlated according to an AR(1) (Autoregressive) process then estimate the correlation of the observed data lagged by one unit as $\rho$ and the variance of the data divided by $(1 - \rho^2)$ is used to determine the true variance of the generating model accounting for the autocorrelation in the data.
- This new variance is the one used in the data generation process.
- In the fit here this is an approximation and the optimal factor could be determined based upon the residuals and model functional form.
- Example

# Example of AR(1) Process - Inflated Variance

| Gate Width | Response |
|---|---|
| 1 | 10.00 |
| 2 | 9.55 |
| 3 | 9.93 |
| 4 | 10.00 |
| 5 | 9.88 |
| 6 | 10.04 |
| 7 | 10.39 |
| 8 | 10.83 |
| 9 | 11.28 |
| 10 | 11.59 |
| 11 | 11.86 |
| 12 | 11.43 |
| 13 | 11.89 |
| 14 | 11.55 |
| 15 | 11.49 |
| 16 | 11.88 |
| 17 | 12.36 |
| 18 | 12.47 |
| 19 | 12.95 |
| 20 | 12.49 |
| 21 | 12.69 |
| 22 | 12.76 |
| 23 | 12.96 |
| 24 | 12.74 |
| 25 | 13.19 |
| 26 | 12.73 |
| 27 | 13.14 |
| 28 | 13.04 |
| 29 | 13.05 |
| 30 | 12.95 |

| | |
|---|---|
| Standard Deviation | 1.19 |
| Variance | 1.41 |

$$Y_\tau = Y_{\tau-1} + \varepsilon_\tau$$

$$Y_1 = 10$$

$$Y_2 = Y_1 + \varepsilon_1 = 10 + -.45$$

$$Y_3 = Y_2 + \varepsilon_2 = 9.55 + .38$$

$$Y_4 = Y_3 + \varepsilon_3 = 9.93 + .06$$

$$\vdots$$

| Gate Width | Response |
|---|---|
| 1 | 10.00 |

| |
|---|
| -0.45 |
| 0.38 |
| 0.06 |
| -0.11 |
| 0.15 |
| 0.35 |
| 0.45 |
| 0.45 |
| 0.31 |
| 0.27 |
| -0.42 |
| 0.46 |
| -0.34 |
| -0.06 |
| 0.39 |
| 0.48 |
| 0.10 |
| 0.48 |
| -0.47 |
| 0.21 |
| 0.07 |
| 0.20 |
| -0.22 |
| 0.44 |
| -0.46 |
| 0.41 |
| -0.09 |
| 0.00 |
| -0.10 |

| | |
|---|---|
| Standard Deviation | 0.32 |
| Variance | 0.10 |

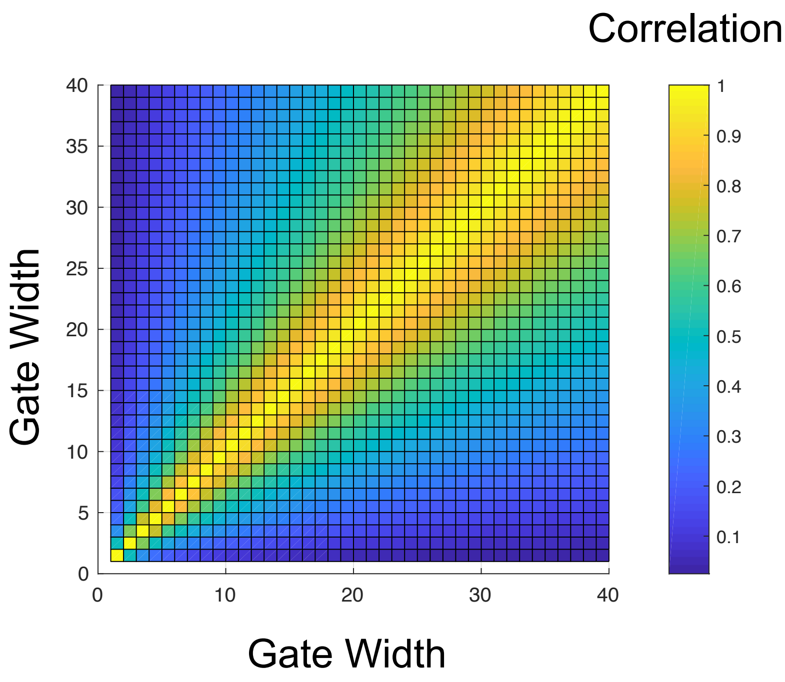
| | Deflation Factors | Deflated Estimates | Observed Error |
|---|---|---|---|
| Standard Deviation | 0.27 | 0.32 | 0.32 |
| Variance | 0.07 | 0.10 | 0.10 |

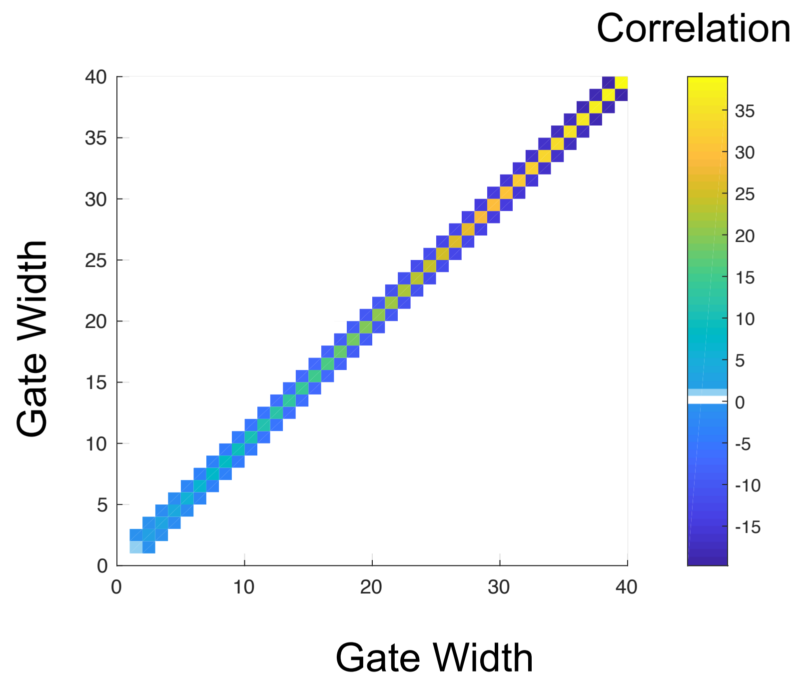
# Neutron Multiplicity: LANL W Covariance Matrix for Curve Fitting

- In neutron multiplicity counting one may fit a curve by minimizing an objective function, $\chi_n^2$.
- The objective function includes the inverse of an $n$ by $n$ matrix of covariances, $W$.
- The inverse of the $W$ matrix has a closed form solution.
- In addition $W^{-1}$ is a tri-diagonal matrix.
- The closed form and tri-diagonal nature allows for a simpler expression of the objective function $\chi_n^2$.
- Minimization of this simpler expression will provide the optimal parameters for the fitted curve.

## Lehmer Matrix n=40

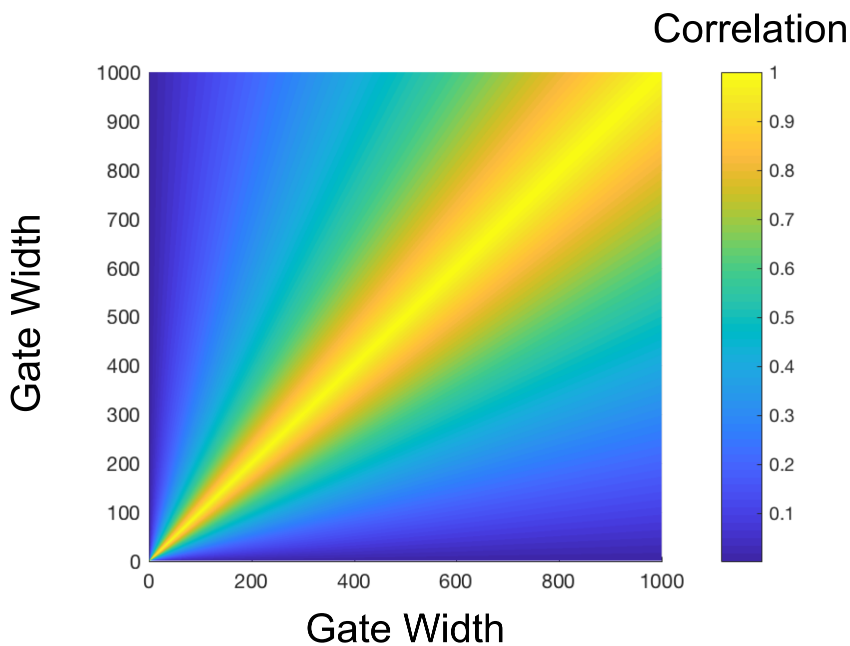
## Inverse Lehmer n=40

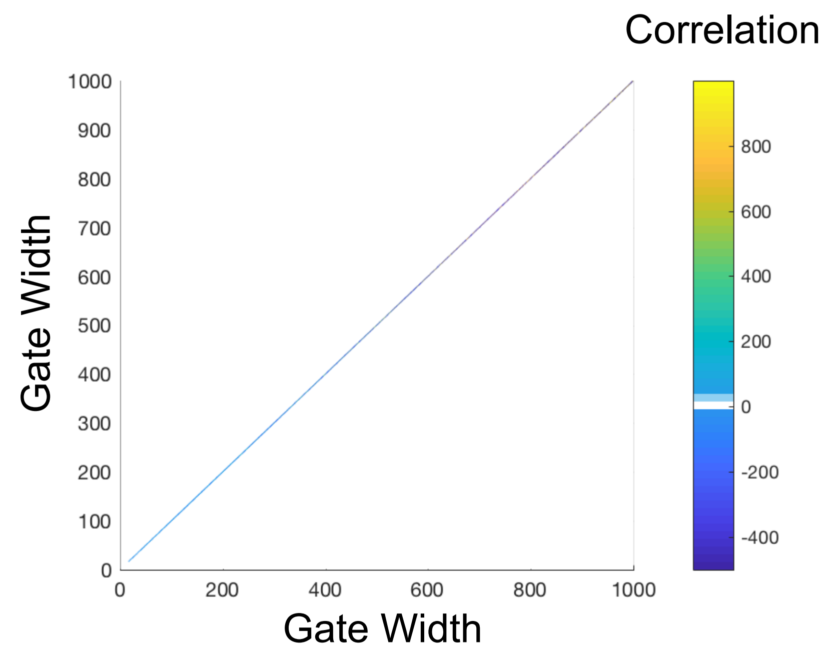


# Lehmer Matrix n=1,000

# Inverse Lehmer n=1,000



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Thank You

Questions?